Bayes Theorem Example

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Conditional Probability

$$(1) \qquad \mathrm{P(C|A)} = \frac{\mathrm{P(A,C)}}{\mathrm{P(A)}}$$

$$(2) \qquad \mathrm{P(A|C)} = \frac{\mathrm{P(A,C)}}{\mathrm{P(C)}}$$

Bayes Theorem

$$(3) \qquad \mathrm{P(C|A)} = \frac{\mathrm{P(A|C)P(C)}}{\mathrm{P(A)}}$$

Let's Make a Deal

Three Doors A, B, C
- One Grand Prize Car

Contestant picks door. Monte Hall opens one of the other doors - booby prize.

Should contestant can stay with original door or switch?

Contestant picks door A,
Monty opens door B; Should switch to C?

Solution: Compare probabilities

grand prize being behind door A with

grand prize being behind door C

Preliminaries:

Probability(Monty opens door B) = P(oB)

Probability(Car in door X) = P(X)

$$\mathbf{P}(\text{Monty opens door } \mathbf{B}) = \mathbf{P}(\text{o}\mathbf{B})$$

$$= \mathbf{P}(\text{o}\mathbf{B}|\mathbf{A})\mathbf{P}(\mathbf{A}) + \mathbf{P}(\text{o}\mathbf{B}|\mathbf{B})\mathbf{P}(\mathbf{B}) + \mathbf{P}(\text{o}\mathbf{B}|\mathbf{C})\mathbf{P}(\mathbf{C})$$
$$= (1/2)(1/3) + (0)(1/3) + (1)(1/3)$$
$$= 1/2$$

$$\mathbf{P}(\text{car in } \mathbf{A}|\text{open } \mathbf{B}) = \frac{\mathbf{P}(\text{o}\mathbf{B}|\mathbf{A})\mathbf{P}(\mathbf{A})}{\mathbf{P}(\text{o}\mathbf{B})} = \frac{(1/2)(1/3)}{1/2} = 1/3$$

$$\mathbf{P}(\text{car in } \mathbf{C}|\text{open } \mathbf{B}) = \frac{\mathbf{P}(\text{o}\mathbf{B}|\mathbf{C})\mathbf{P}(\mathbf{C})}{\mathbf{P}(\text{o}\mathbf{B})} = \frac{(1)(1/3)}{1/2} = 2/3$$

Answer: Yes, Switch as the probability that the car is in $\mathbf{C}$ is greater

Simpler Way

Think of it as two choices -

Choice 1) Door A has a probability of 1/3 having the car behind it. (as each of the doors are equally likely)

Choice 2) Not Door A has a probability of 2/3. Monty puts the full 2/3 probability on the exact door to switch over to.

2

This is the first slide.

You can typeset *Emphasized text*.

You can also typeset **Bold**, *Italics*, *Slanted* and `Typewriter` text. Roman fonts are not available.

Point size can be changed by making use of the tiny, scriptsize, footnotesize, small, normalsize, large, Large, LARGE, huge and Huge commands.

3 This is the second slide.

The numbered equation

$$u_{tt} - \Delta u + u^5 + u\,|u|^{p-2} = 0 \text{ in } \mathbf{R}^3 \times [0, \infty[\,. \tag{4}$$

is automatically numbered as equation 4.